Number Sense Assessment Numbers 0-20 In *Teaching Student Centered Mathematics PreK-2,* Van de Walle, et al, describes four relationships that need to be developed for students to gain number sense with numbers up to 20 so they not only can operate with those numbers, but also understand them:

- Spatial relationships recognizing how many without counting by seeing a visual pattern.
- 2) One and two more, one and two less this is not the ability to count on two or count back two, but instead knowing which numbers are one and two less or more than any given number.
- 3) Benchmarks of 5 and 10 ten plays such an important role in our number system (and two fives make a 10), students must know how numbers relate to 5 and 10.
- 4) Part-Part-Whole seeing a number as being made up of two or more parts.

How to use this assessment:

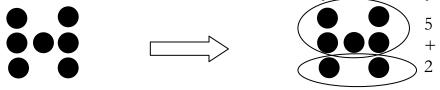
Each of the four relationships is explained in more detail to give you more insight into each relationship before you administer the assessment. So read those first. Also, read the "Administering the Assessment" page before you give the assessment. Then as you give the assessment write brief notes that will allow you to determine what level each child is at for the items. If a child is in the Level 1 or 2 for most items for that relationship, then it is an area that child needs to work on developing. We want to get children to the top level in each assessment item.

Check out my blog to post your comments and questions about the assessment here:

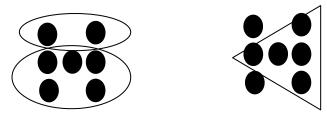
 $\underline{http://www.therecovering traditional ist.com/number-sense-assessment/}$

The Power of The Relationship: Spatial Relationships

Close your eyes and think of "seven." What did you see? The numeral '7'? The word *seven*? Or are you one of the few that has a picture of seven things? For many children the only picture they have of seven is the numeral. That "picture" makes it almost impossible for them to see how that can be broken into a 5 and a 2, or a 6 and a 1. However if a child sees this for seven, they <u>can</u> actually see a 5 and a 2:



Using that same pattern, some children may see the 5 and 2 differently:



And even cooler is that they do not have to see it as only 5 and 2. Some may see 6 and 1 or 4 and 3 (Can you see them?). The point is to give them a visual representation of the abstract symbols we ask them to compute with. Many children are still grappling with making sense of single digits; that 7 can be 7+0, 6+1, 5+2, 3+4, and so on. However at the same time they are dealing with this understanding we are asking them to think about combining those same numbers but with place value: 1 and 6 now become 16. When we give them a visual picture of 1 + 6 they can see for themselves that it makes 7 and not just because we put a '+' sign in between them.

Building spatial relationships also builds the idea of subitizing. Subitizing is being able to instantly recognize how many are in a set (Clements, 1999). For most people we can only subitize small sets (less than 5) unless they are in some kind of familiar pattern. For larger sets of numbers we tend to decompose the set into smaller sets that we do instantly recognize. Look at the dots below and determine how many total dots there are:

Did you do it without having to count every single dot to make sure of your total? Now try this one:

With this pattern you can instantly recognize the familiar pattern of five and there are two of them, so there has to be ten. Children should also have practice with subitizing using their fingers. When children are in the counting stage and are trying to solve 6 + 7, they hold the 6 in their head and then count on 7. But how do they know when to stop counting? As they are putting up fingers they have to be able to 'see' when they have 7 fingers up without having to count them. Some children have to count out seven fingers before they can use those fingers to count with.

Dot patterns and fingers are not the only way to build spatial relationships. A number's placement along a number path helps give children a visual picture as well. By showing the placement in a number path children will start to understand the magnitude of a number; 9 is much larger than 2, but 5 and 6 are close to each other. A number path is a powerful tool for children which leads into a number line that children can use even as they progress into upper elementary and into their learning of fractions.

The ten-frame and the Rekenrek (aka <u>MathRack</u>) are other tools that build spatial relationships, but they also help children with the benchmarks of 5 and 10. So more information about those are in that section of this assessment.

Note: If you do not have dot pattern cards, you can download ones that I have created by clicking <u>HERE</u>.

Spatial Relationship	Assessment Checklist:	Date Given	

	Child 1	Child 2	Child 3	Child 4	Child 5
1. When asked to represent 4 using their fingers, the child produces 4 fingers without having to count them.					
2. When shown the familiar dice dot patterns for 1 through 6, the child instantly tells how many without having to count each individual dot. (use dot pattern cards)					
3. When shown a dot pattern card for 3 seconds, the child can reproduce the same number with counters, their fingers, dot stickers, or some other manipulative.					
4. When asked to represent 8 using their fingers, the child produces 8 fingers by either instantly showing 8 or by showing 5 then counting 6, 7, 8. (indicate in the box which strategy they used to show 8)					
5. When shown a dot pattern card larger than 6, the child can determine the number without having to count each individual dot/item.					

The items that say "when shown a dot pattern card," you do not have to only use the dot pattern cards. Use the rekenrek or ten frame cards to vary the spatial picture that the child sees. This is a way to make sure they can recognize an 8 (or any other number) whether it is a dot pattern, in the ten frame, or on a rekenrek.

Assessment Task	What instructor says	Possible child responses
1. When asked to represent 4 using their fingers, the child produces 4 fingers without having to count them.	"Show me four using your fingers."	Level 1: Child does not show 4 fingers Level 2: Child puts one finger up at a time saying "1,2,3,4" and shows 4 Level 3: Child puts all four fingers up at once without having to count individually
2. When shown the familiar dice dot patterns for 1 through 6, the child instantly tells how many without having to count each individual dot. (do use dot pattern cards for this assessment item)	Flip over a dot pattern that contains the dice pattern for any number 1-6 and ask the child, "How many dots are on this card?"	Level 1: Child does not give an accurate response Level 2: Child counts each dot one-byone to get the correct answer Level 3: Child instantly recognizes the amount without counting and says the correct number within 3 seconds
3. When shown a number using a dot pattern card (you can use a ten frame or rekenrek also) for 3 seconds, the child can reproduce the same number with counters, their fingers, dot stickers, or some other manipulative.	"I am going to show you a picture and I want you to use your fingers or these counters to create the same number of items you see in the picture."	If the child does not start working you can prompt them by saying "how many dots did you see?" Level 1: Child cannot produce the same number Level 2: Child counts one-by-one to get the correct number Level 3: Child instantly produces the same number of items without counting
4. When asked to represent 8 using their fingers, the child produces 8 fingers	"Show me eight using your fingers."	Level 1: Child does not show 8 fingers Level 2: Child puts one finger up at a time saying "1,2,3,4" until he/she has 8 Level 3: Child puts up all five on one had without counting, but then counts "6,7,8" on the other hand Level 4: Child puts all 8 fingers up at once without having to count any
5. When shown a dot pattern card (or ten frame/rekenrek) containing a number larger than 6, the child can determine the number without having to count each individual dot/item.	"I am going to show you a picture and I would like you to tell me how many dots/items you see."	Level 1: Child does not give the correct number Level 2: Child counts each dot/item one- by-one to get the correct answer Level 3: Child instantly recognizes the amount without counting and says the correct number within 3 seconds

The Power of The Relationship: One/Two More and Less

Knowing one/two more or less allows students to be flexible thinkers and aides in mental computation. For instance if a child understands that 9 is one less than 10, when they see 9+5 they can think to themselves "That is like 10+5, which is 15 so it is just one less." Not understanding this relationship limits students to seeing 9+5 as just 9+5, and if they do not remember that "fact" they revert back to counting on their fingers. This then leads into the same kind of calculations when dealing with multidigit addition where all too often we see children still counting on their fingers instead of using relationships. When a child is presented with a problem like 59+25, we often see them adding 9+5 on their fingers then adding 5+2+1. If instead children have the relationship built of One/Two More and Less, it is faster, (and dare I say easier) than the traditional algorithm, to think of the problem as 60+25 and then take one away.

On the next page is the addition fact chart. If we teach the addition facts as isolated facts (learn +0, +1, +2, +3...), children have 121 facts to remember. If instead we focus on four types of facts and on building the relationship of One/Two More and Less we cover all 121 'facts'. The larger benefit of allowing children to learn their facts in this way is that it becomes so much easier for children to pull facts out of their memory. Try to memorize this eleven digit number: 25811141720. Do you think you have it? You might for right now, but will you be able to remember it in three days, how about two months? If instead I tell you to look for a relationship within the numbers, can you remember them now? What if I tell you that if you start with the first number and add 3 you get the next number and keep adding 3; can you remember all the numbers now? I have done this activity with a lot of teachers and it is amazing to see that they remember this eleven digit number months after I first give it to them. When we focus on how things are connected it becomes easier to retrieve them from our memory. This is the way child learn facts; some can memorize and do just fine, others see relationships between facts, and others cannot memorize and do not see relationships so we have to explicitly teach those relationships to them. The four types ©Mathematically Minded, LLC www.TheRecoveringTraditionalist.com

of facts to focus on are highlighted in the chart, the dark color is the 'fact' and the lighter version of that color is +/- one or two:

- Orange doubles
- Blue 10 plus something
- Green facts that make 10
- Purple plus zero

+	0	1	2	3	4	5	6	7	8	9	10
0	0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10	11
2	2	3	4	5	6	7	8	9	10	11	12
3	3	4	5	6	7	8	9	10	11	12	13
4	4	5	6	7	8	9	10	11	12	13	14
5	5	6	7	8	9	10	11	12	13	14	15
6	6	7	8	9	10	11	12	13	14	15	16
7	7	8	9	10	11	12	13	14	15	16	17
8	8	9	10	11	12	13	14	15	16	17	18
9	9	10	11	12	13	14	15	16	17	18	19
10	10	11	12	13	14	15	16	17	18	19	20

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Another issue children sometimes struggle with is a big idea known as hierarchical inclusion. This is the idea that if I have 7 of something I also have 6, 5, 4, etc; 7 includes all the numbers below it. You can observe this when you ask a child questions like, "We have 6 cupcakes and 4 kids. Do I have enough cupcakes so that each child can have one?" Some children do not understand that 6 includes 4, but that idea will start to be developed in these lessons.

One/Two More and Less Assessment Checklist:	Date Given
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	Child 1	Child 2	Child 3	Child 4	Child 5
When shown a numeral, like 7, and asked "what number is one more than this number?" the child answers correctly. (if they can do this use another number and ask what number is two more)					
Ask the child to count out 5 items. After they have successfully done that, add one item to their pile and ask how many they have now. Does the child know there are 6 without recounting them all? (if they can do this ask a similar question but add TWO)					
When shown a numeral, like 6, and asked "what number is one less than this number?" the child answers correctly. (if they can do this use another number and ask what number is two less)					
Ask the child to count out 9 items. After they have successfully done that, take one item away and ask how many they have now. Does the child know there are 8 without recounting them all? (if they can do this ask a similar question but take TWO away)					

For each item a specific number is given, but you can use **smaller numbers if the child does not get it right** the first time or you can **extend the assessment by doing the same activities with numbers in the teens.**

Assessment Task	What Instructor Says	Possible child responses
When shown a numeral, like 7, and asked "what number is one more than this number?" the child answers correctly. (if they can do this use another number and ask what number is two more)	Flip over a 3x5 card that has a numeral written on it (5-9) and say "What number is one more than this number?" If the child answers correctly flip over the next number and ask "What number is two more than this number?" Notate in the checklist if they can do +1 and +2	Level 1: Child will give an incorrect answer (try a smaller numeral) Level 2: Child can give correct answer for +1 Level 3: Child can give correct answer for +1 and +2 Level 4: Child can give correct answer when presented with a numeral in the teens
Ask the child to count out 5 cubes (or any item) from a larger group. After they have successfully done that, add one cube to their pile and ask how many cubes they have now. Does the child know there are 6 without recounting them all? (if they can do this ask a similar question but add TWO)	"Please count out 5 cubes." If the child cannot count 5 cubes, do it together so they see the 5 cubes being counted. Then add one cube to their pile and say, "I added one cube to your pile, how many do you have now?" If they answer correctly say, "Please count out 7 cubes now I added two cubes to your pile, how many cubes are there?"	Level 1: Child answers incorrectly Level 2: Child gives the correct number of cubes but has to recount all the cubes. Level 3: Child knows how many when you add one without having to recount. Level 4: Child knows how many when you add one, but recounts when you add two. Level 5: Child knows how many when one or two are added.
When shown a numeral, like 6, and asked "what number is one less than this number?" the child answers correctly. (if they can do this use another number and ask what number is two less)	Flip over a 3x5 card that has a numeral written on it (5-9) and say "What number is one less than this number?" If the child answers correctly flip over the next number and ask "What number is two less than this number?" Notate in the checklist if they can do -1 and -2.	Level 1: Child will give an incorrect answer (try a smaller numeral) Level 2: Child can give correct answer for -1 Level 3: Child can give correct answer for -1 and -2 Level 4: Child can give correct answer when presented with a numeral in the teens
Ask the child to count out 9 cubes (or any item). After they have successfully done that, take one cube away and ask how many cubes they have now. Does the child know there are 8 without recounting them all? (if they can do this ask a similar question but take TWO away)	"Please count out 6 cubes." If the child cannot count 6 cubes, do it together so they see the 6 cubes being counted. Then remove one cube from their pile and say, "I took away one cube from your pile, how many do you have now?" If they answer correctly say, "Please count out 9 cubesnow I took away two cubes from your pile, how many cubes are there?"	Level 1: Child answers incorrectly Level 2: Child gives the correct number of cubes but has to recount all the cubes. Level 3: Child knows how many when you remove one without having to recount. Level 4: Child knows how many when you remove one, but recounts when you remove two. Level 5: Child knows how many when one or two are removed.

The Power of The Relationship: Benchmarks of 5 and 10

The numbers 5 and 10 are important benchmarks in our number system because we use a Base 10 (and 2 fives make a ten). Helping children see how numbers relate to 5 and 10 becomes useful as they start to compute with numbers. If you know that 7 is 5 + 2 or it is three less than 10, think about how that could help children as they solve these problems: 7+8, 5+2, 7-2, 7-3, 13-7, 7+3, 57+3, 68+7, 47+6 (Van de Walle, 2014). How would you use the benchmarks of 5 and 10 to help you solve those problems? Here are two examples:



Those were just one way to use 5 and 10 to solve these problems; there are other ways as well. Children tend to receive lots of experiences decomposing numbers but what is lacking is knowing how that knowledge can help them and which way they should decompose a number. For example I may know all the ways to decompose the number 6 (5+1, 4+2, 3+3), but when I am presented with the problem 6 + 9 do I know how decomposing the 6 can be helpful and which decomposition of 6 I should use to help me solve the problem? If I understand that I can break apart the 6 and I know 9's relationship to 10, I can instantly see that I should decompose the 6 into a 5 and 1 instead of a 4 and 2 (or 3 and 3).

It is a large leap for some students to get from the counting phase to the derived fact stage because making that leap means the child can group numbers instead of counting one-by-one and they know which grouping to use (Carpenter, et al, 1999):

A child who is in the counting phase solves 8 + 7 like this; they hold "8" in their head then to add 7 they count one-by-one, "9, 10, 11, 12, 13, 14, 15."

A child who can use derived facts may still start with the 8, but uses groups to add the 7; "8 plus 2 is 10. I know 7 can be a 2 and 5. I've added 2, so I need to add 5 more which makes it 10 + 5...15."

There is a lot of number sense that children need in order to use derived facts. Many children still do not see groups within a number, to them 7 is just 7 (1,2,3,4,5,6,7) it is not a 5 and a 2 and they do not know that it is three away from 10. The activities in this section help children see groupings of five and ten. The Spatial Relationships section helps children see all types of groupings for a number which is important, but the groupings of five and ten are extremely important. One of the most common derived facts we try to help children understand is the "make a ten" strategy. However, if they do not know a number's relationship to 10, they cannot use the "make a ten" strategy.

Even if a child can "make a 10" does not mean they will use it as an efficient strategy. For example, with the 8 + 7 problem, the child might have been able to break the 7 into a 2 and 5 to make 8 + 2 + 5. However, this is only helpful if they know 10 + 5. Children who do not know the "10 plus something" idea will count on one-by-one to add 5 to the 10. Time needs to be spent helping children understand the numbers 11 through 19 as "10 plus something." This idea is the beginning of helping children understand place value. To young children they do not see the "1" as being any different just because we moved it over a centimeter; think of the number "11," why is one of the "1s" any different from the other??? Children need experiences with seeing objects grouped into ten and some more (example: 17 cubes separated into 10 cubes and 7 cubes) and relating that to the written form of the number.

Note: The assessment for this section uses a Rekenrek (aka MathRack). If you do not have a Rekenrek I work closely with the MathRack company and highly recommend their product (www.mathrack.com). However, you can also use my directions on how to build your own DIY Rekenrek by downloading the instructions HERE.

Benchmarks of 5 and 10 Assessment Checklist:	Date Given
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	Child 1	Child 2	Child 3	Child 4	Child 5
Give the child a Rekenrek. Ask them to show you 7 beads on the Rekenrek. Watch to see how the child counts out the 7.					
Count out 6 items and ask the child how many more cubes are needed to make 10.					
Count out 17 items and ask the child how many cubes you would need to take away to make 10.					
Ask the child to count out 12 items and write the number on paper. Then ask them to show you with their items what the '2' (point to it) and the '1' means with their items they counted.					

Note: The assessment for this section uses a Rekenrek (aka MathRack). If you do not have a Rekenrek I work closely with the MathRack company and highly recommend their product (<u>www.mathrack.com</u>). However, you can also use my directions on how to build your own DIY Rekenrek by downloading the instructions <u>HERE</u>.

Assessment Task	What Instructor Says	Possible child responses
Give the child a Rekenrek. Ask them to show you 7 beads on the Rekenrek. Watch to see how the child counts out the 7.	"I'm giving you a tool we will be using later in class. This tool has beads on it, please slide over 7 beads on one of the rows?"	Level 1: cannot count 7 beads Level 2: counts the 7 beads one-by- one Level 2: grabs 5 beads and 2 more Level 3: counts three beads to leave and pushes over the rest
Count out 6 cubes and ask the child how many more cubes are needed to make 10.	Count out 6 cubes in front of the child, say to the child "I have 6 cubes here, how many more do I need put out here so that I have 10 cubes?" If child cannot do this task use a number path, put your finger on 6 and ask them how many jumps you would have to do to get to the 10 spot.	Level 1: child cannot do the task Level 2: child adds cubes one-by- one to get to 10, then counts how many they added Level 3: child adds groups to get to ten (add 2, then 2) Level 4: child just knows you need to add 4
Count out 17 cubes and ask the child how many cubes you would need to take away to make 10.	Count out 17 cubes in front of the child, say to the child "I have 17 cubes here, how many do I need to take away so that I have only 10 cubes?" If child cannot do this task use a number path, put your finger on 16 and ask them how many jumps you would have to go back to get to the 10 spot.	Level 1: child cannot do the task Level 2: child takes away cubes one-by-one to get to 10, then counts how many they took away Level 3: child takes away groups to get to ten (take away 5, then 2) Level 4: child just knows you need to take away 7
Ask the child to count out 12 cubes and write the number on paper. Then ask them to show you with their cubes what the '2' (point to it) and the '1' means with their cubes.	"I have a bunch of cubes here; will you count out just 12 cubes for me, please?" "Great, now on this paper please write that number as big as you can." "So this number tells me how many cubes you have right here?" (wait for confirmation from the child) Then point to the 2 and say "Show me with your cubes what this number means. Put the cubes right here next to the number." "Okay, can you show me what the 1 (point to it) means?"	Level 1: child cannot show the two numbers with the cubes Level 2: child shows 2 cubes for the "2" and but 1 cube for the "1" Level 3: does level 2, but when asked about the remaining 9 cubes, the child figures out those go with the 1 to make a 10. Level 4: child shows the 2 cubes and the ten that represent the 1.

The Power of The Relationship: Part-Part-Whole

The three prior relationships help build the Part-Part-Whole concept; the idea that a number (7) is not just seven items; it can be decomposed into many different parts. Early on, children use this concept with the numbers up to 10; 7 can be 6 + 1, 5 + 2, etc. As their mathematical education continues this idea is useful as they start to deal with multidigit numbers (37 can be 30+7, 32+5, 20+17, etc.) and fractions (7 is the same as $6 \frac{1}{2} + \frac{1}{2}$ or $5 \frac{3}{4} + 1 \frac{1}{4}$ and so on) and on into algebraic reasoning (x + 5 = 7).

A child can become accurate and efficient with basic facts through memorization. However, that knowledge can only be applied to those particular tasks (Baroody, et.al, 2009). A child who has only memorized 8 + 7 = 15 does not see the connection to the problem 38 + 7. When we focus on the relationships in this book it helps give children flexibility when dealing with their basic facts and extending that knowledge to new tasks. Children who can decompose numbers, understand a number's relationship to 5 and 10, and know one/two more and less will see both 8 + 7 and 38 + 7 in the same way:

Child 1: "8 + 2 makes 10 and add 5 more makes 15."

"38 + 2 makes 40 and add 5 more makes 45."

Child 2: "8 + 7 is like 7 + 7 which is 14, plus one more makes 15."

"38 + 7 is like 37+ 7 which is 44, plus one more makes 45."

Child 3: "8 + 7 is like (5 + 3)+(5 + 2). I put the two 5s together to make 10 and add the 3 and 2 gives me 15."

"38 + 7 is like (35 + 3)+(5 + 2). I put the 35 and 5 together to make

40 and add the 3 and 2 to give me 45."

All of these strategies should be accepted and encouraged. When we build a child's number sense it promotes *thinking* instead of just *computing*. Too many children (and even adults) compute without thinking, thus they do not notice when an answer is unreasonable.

The most common error with multidigit addition is when a child does not 'carry the one,' they just add and write the numbers down:

29 +17 316

If a child *thinks* before they *compute*, they might think to themselves "29 is just one away from 30. If I add one to 29 (to make 30) I need to add 16 more. So it is 30 + 16 which is 46." In order to think that way a child must understand a number's part-part-whole relationship; that a number can be decomposed into parts and will still stay the same.

This idea can be a large leap for some students. Basically we are asking children to think of 8 + 7 as the same as 8 + 2 + 5. For a child who believes 7 is just 7, they do not understand how that 7 became a 2 and 5 or that separating the 7 into a 2 and 5 did not change the fact that they still have seven total. Children may have no problem seeing that they need to add 2 to the 8 to make 10, but they may have a problem recognizing the result of taking away 2 from the 7 (Baroody, et.al, 2009). That is why this section of the book emphasizes decomposing numbers into their parts. The more practice children have with that concept, in meaningful and fun ways, the more automatic it becomes.

Part-Part-Whole Assessment Checklist:	Date Given
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	Child 1	Child 2	Child 3	Child 4	Child 5
Count out 7 items, separate into two groups (like a 3 and 4). Does the child know there are still 7 items?					
Count out 8 items. Put some of those items in a cup so the child cannot see them. The child can see how many are still left, ask them how many you put in the cup.					
Using one of the dot pattern cards, cover some of the dots with a Post-it note so that they are hidden from the child. Tell them how many dots are on the entire card, but some are covered by the Post-it. Can they determine the number of hidden dots?					

Assessment Task	What Instructor Says	Possible child responses
Count out 7 cubes (or any item), separate into two groups (like a 3 and 4). Does the child know there are still 7 cubes?	I am going to put out 7 cubes on the table. Count them out loud so the child can hear you. How many cubes did I just count out? Wait for the child to tell you 7. I am going to put some of the cubes over here on this paper. How many cubes do I have out on the table?	Level 1: child believes there is not 7 cubes Level 2: child has to count them to determine if there are still 7 Level 3: child knows that even though you moved some of them, there are still 7
Count out 8 cubes (or any item). Put some cubes in a cup so the child cannot see them. The child can see how many are still left, ask them how many you put in the cup.	I am going to count out 8 cubes. Count them out loud so the child can hear you. Now, I am going to put some of them in this cup to hide from you. Grab 3 in your hand so the child cannot see how many you took and put them in the cup. Now we had 8, there are only these cubes left. How many cubes did I put in the cup?	Level 1: child cannot determine the amount you hid in the cup. (Try a smaller number, like 4, and see if they can determine how many you hide.) Level 2: child counts one-by-one to figure out how many you hid. Level 3: child can tell you within 3 seconds how many you hid.
Using one of the dot pattern cards, cover some of the dots with a Post-it note so that they are hidden from the child. Tell them how many dots are on the entire card, but some are covered by the Post-it. Can they determine the number of hidden dots?	This card has 6 dots on it, but some of the dots are covered by this Post-it note. Can you tell me how many dots are covered by the Post-it?	Level 1: child does not give an accurate answer. (Try a smaller number, like 4, and see if they can determine how many are covered.) Level 2: child counts one-by-one to figure out how many dots are hiding under the Post-it. Level 3: child can tell you within 3 seconds how many dots are hiding under the Post-it.